Tripartite Line Tracks – Bipartite Line Tracks

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Abstract. Theories of shapes are important for object recognition and for reasoning about the behaviour of objects, both tasks strongly constrained by shape. Whereas the extraction of shape properties has extensively been studied in vision, there is still a lack of qualitative shape descriptions which allow reasoning about shapes with AI techniques in a flexible manner.
In this paper we present a qualitative shape description. This description is based on a set of qualitative relations which can be combined to construct arbitrary polygonal shapes. As we are interested in demonstrating how qualitative reasoning approaches can be applied to shape descriptions, our theory is confined to stylised shape representations which are obtainable by applying conventional image processing techniques. We will show how to qualitatively reason about shapes.

1 Introduction

Shape descriptions have extensively been studied in vision. Traditionally, such descriptions resort to quantitative methods that often give rise to high dimensional feature spaces which are useful for classification tasks. However, these representations are inappropriate for symbolic reasoning and do not provide for dealing flexibly with shapes. But this becomes important in more sophisticated reasoning tasks when studying the behaviour of objects, for example, their change in shape, motion in space, and interaction with other objects.

Qualitative abstractions allow reducing information in order to obtain descriptions which are more tractable than quantitative descriptions. A qualitative shape description is investigated which represents a kind of qualitative abstraction which permits application of reasoning techniques to shapes that have been devised for qualitative spatial reasoning, especially in the context of navigation.

For any qualitative shape description, there exist several conceivable differences to be considered. Shape descriptions can be classified as either two-dimensional or three-dimensional, as either can be related to regions or just confined to outlines. These may loosely be the two most important distinctions. For an overview of qualitative shape approaches see [2]. Our description will be restricted to two-dimensional polygonal outlines since we are mainly interested in demonstrating how shape descriptions can be devised by considering orientation information and navigation tasks. For that purpose, we will confine ourselves to one single approach from Freksa and Zimmermann which is described in [5], and which we will adopt to shape reasoning problems.

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2 Obtaining Shapes by Navigation

With qualitative descriptions, objects are often regarded in relation to other objects. This concerns particularly navigation tasks where the position of objects are described with respect to landmarks. One example is a simple localisation task, stated by [5]:

> *Walk down the road. You will see a church in front of you on the left. Before you reach the church turn down the path that leads forward to the right. The question one might ask on his way down the path is, where is the church with respect to me?*

When recording the path someone walks along we obtain a trajectory. Any trajectory can simultaneously be accounted for being part of a shape's outline. Necessarily, we will get something of the kind of a shape description by considering how the localisation task can be solved. That is to say, we will learn something about properties of trajectories which are made up of landmarks and points between which movements take place. More precisely, we are interested in the representation which has been used to solve such tasks, rather than in the solution itself.

The representation that Freksa and Zimmermann have introduced is based on what they call an *orientation grid*. This grid is aligned to the orientation determined by two points, i.e. the start-point and the end-point of a movement (see Fig. 1.a). A third point can then be considered to be on the left, on the right, or straight on this movement-vector, and furthermore, it can be before the start-point, behind the end-point, or in between (see Fig. 1.b). Eventually, it can be on one of the two lines separating the three consecutive regions (the dotted lines in Fig. 1.b). We are then able to distinguish fifteen different positions, as shown in Fig. 1.c, and the localisation task can now simply be stated by asking where a landmark c is with respect to a vector $a \to b$ (see Fig. 1.d).

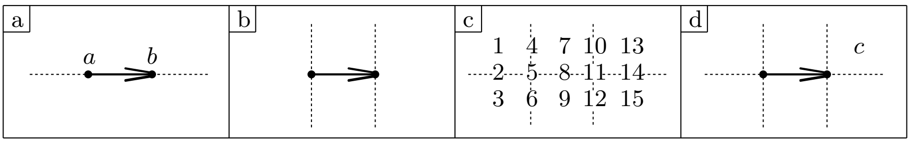

Fig. 1. The orientation grid introduced by one continuously drawn vector distinguishes fifteen positions

Freksa and Zimmermann demonstrate how to reason about positions with respect to landmarks by using the orientation grid. In contrast, we will consider different possible trajectories put up by means of landmarks, and we will show how to describe the outlines of shapes with the aid of the orientation grid.

3 Bipartite Line Tracks

We will introduce our approach for qualitatively describing shapes. For this description, the orientation grid is adapted in order to represent shape primitives. These primitives are made up of two connected lines according to the localisation task which was described in the previous section. In addition to a movement-vector, a line is considered which connects the end-position of this movement to a landmark. In this way, each primitive corresponds to a bipartite path which is described by the start-position of a movement, a, its end-position, b, and additionally by a landmark, c.

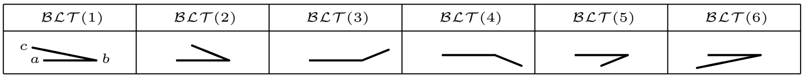

Fig. 2. Six distinguishable classes of Bipartite Line Tracks; the orientation of the horizontal line corresponds to the vector $a \to b$

In order to construct shape primitives only general positions are considered, i.e. the six positions which do not lie on the orientation grid: $1, 3, 7, 9, 13$, and 15 in Fig. 1.c. The other nine positions lying directly on the orientation grid are called singular positions. We will discuss later how to deal with singular positions. As there exist six general positions with respect to a vector $a \to b$, we obtain six bipartite relations, shown in Fig. 2. We will call these primitives *Bipartite Line Tracks*, and the i-th relation can be accessed by $\mathcal{BLT}(i)$, for $i \in \{1, 2, 3, 4, 5, 6\}$.

3.1 Polygons

A closed polygon with $k \geq 2$ lines is described as a vector of k $\mathcal{BLT}$s, an open polygon as a vector of $k-1$ $\mathcal{BLT}$s: $\mathcal{BLT}(i, j, ...), i, j \in \{1, 2, 3, 4, 5, 6\}$. In order to be able to treat each arbitrary polygon with k lines, and not only those which consist of a multiple of two lines, a vector of $\mathcal{BLT}$s describes a polygon in such a way that two consecutive $\mathcal{BLT}$s share one line.

For closed polygons, different $\mathcal{BLT}$ descriptions will be obtained depending on where one starts to enumerate a polygon. Any description can be converted into another equivalent description by means of a cyclic permutation of the $\mathcal{BLT}$s involved. For a given polygon we choose the description that comes first in the ordering with respect to the $\mathcal{BLT}$ numbers. We define an anticlockwise orientation regarding the two-dimensional plane, i.e. we describe any contour anticlockwise. Fig. 3 shows an example.

3.2 Neighbourhood Graph

Fig. 4 shows the neighbourhood graph adopted from [4]. In this graph, two relations are connected if they are just separated by one of the lines of the

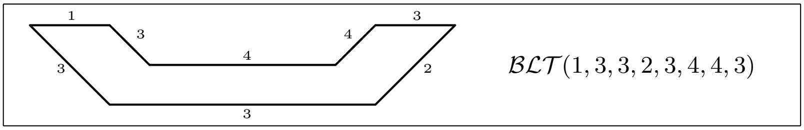

Fig. 3. Example of a polygon described by Bipartite Line Tracks anticlockwise

orientation grid. In other words, two relations are neighbours if one relation can be transformed into the other relation by continuously moving one endpoint to another position whilst crossing the orientation grid exactly once. This graph will be used for reasoning tasks, and at first we will show how to deal with singular positions with the aid of the neighbourhood graph.

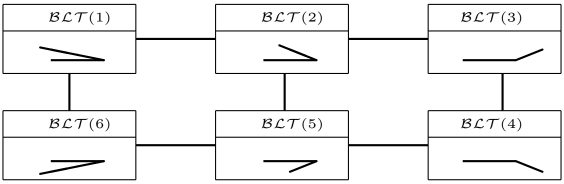

Fig. 4. Neighbourhood graph of the $\mathcal{BLT}$-relations

3.3 Singular Line Tracks

To be more precise, $\mathcal{BLT}$s which have been considered so far would have been better denominated as *General Bipartite Line Tracks* because singular positions were disregarded. Line Tracks that comprise inter alia singular positions are of special interest with respect to the boundaries of artificial objects which often have perpendicular sides. Orthogonality corresponds to relations in singular position.

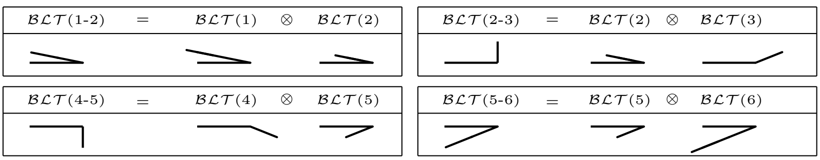

Fig. 5. Singular Bipartite Line Tracks

A *Singular Bipartite Line Track* is referred to as $\mathcal{BLT}(a\text{-}b)$, i.e. $\mathcal{BLT}(a\text{-}b)$ refers to an edge in the neighbourhood graph, whereby a and b are the relations which are connected by this edge. As an example, consider $\mathcal{BLT}$(1-2) in Fig. 5. This singular position is in between the two general positions $\mathcal{BLT}(1)$ and $\mathcal{BLT}(2)$.

Singular line tracks with the endpoints lying on the positions 2, 5, 8, 11, or 14 (see Fig. 1) are in accordance with degenerated cases where we only perceive a single line. These positions are denoted in the following way: $\mathcal{BLT}(3-4)$ (position 14 in Fig. 1.c), $\mathcal{BLT}(2-3-4-5)$ (position 11), $\mathcal{BLT}(2-5)$ (position 8), $\mathcal{BLT}(1-2-5-6)$ (position 5), and $\mathcal{BLT}(1-6)$ (position 2). This notation allows us to consider all those general positions between which there is uncertainty regarding the position of a point or line, respectively. Uncertainty arises whenever one perceives any position somewhere near the transition between two or more general positions. In most cases, a singular position will not definitely be recognised as beeing singular. One will simply have doubts regarding a general position which lies near the transition to another general position. All in all, singular positions are mainly treated in a way that is related to the question of how one has to cope with uncertainty regarding the positions near boundaries of two or more neighbouring regions.

3.4 Non-oriented Primitives

With Bipartite Line Tracks we introduced a shape description which consists of six oriented shape primitives. Oriented primitives allow us to discriminate contour parts that are oriented to the shape of an object from those which are mirror-symmetrical to the former parts, i.e. parts which are oriented to the background. Hence, we distinguish $\mathcal{BLT}(1)$ and $\mathcal{BLT}(6)$, $\mathcal{BLT}(2)$ and $\mathcal{BLT}(5)$, as well as $\mathcal{BLT}(3)$ and $\mathcal{BLT}(4)$ (see Fig. 4). In ambiguous cases we write more precisely $\mathcal{BLT}_6$ in order to refer to these six relations.

Fig. 6. Different polygons described by $\mathcal{BLT}_3(3,3)$, thus not distinguishable by $\mathcal{BLT}_3$

In the context of vision we often have to deal with incomplete shape information. As a consequence, partial shape information has to be described. But frequently nothing can be said concerning the orientation of shape parts since in most instances it is not known on which side of any contour-part the figure or the ground is. As such, it seems reasonable to consider primitives which are not oriented and we obtain only three relations, i.e. one obtuse angled primitive, like $\mathcal{BLT}_6(3)$, and two kinds of acute angled primitives, like $\mathcal{BLT}_6(1)$ and $\mathcal{BLT}_6(2)$. These three relations will be referred to as $\mathcal{BLT}_3$. Such non-oriented primitives have the disadvantage that it is not possible to distinguish some shapes anymore, as, for example, the two polygonal parts in Fig. 6. In order to distinguish more complex non-oriented primitives line tracks made up of three lines will be considered in the next section.

4 Tripartite Line Tracks

We now consider three end-point-connected lines, also described by the orientation grid, as shown by the example on the right of Fig. 7. The medial line determines the orientation grid and the two endpoints are described with respect to the medial line by considering their position with regard to the orientation grid.

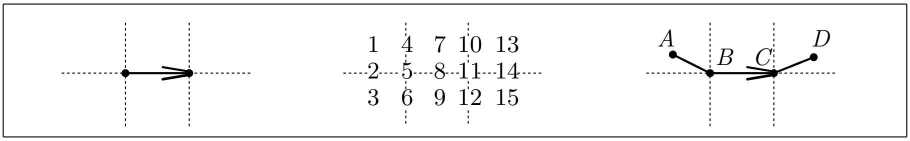

Fig. 7. Left: The orientation grid introduced by one continuously drawn vector, distinguishes fifteen positions (Middle); Right: A line track ($\overline{AB}, \overline{BC}, \overline{CD}$) which consists of three connected lines

4.1 $\mathcal{TLT}_{36}$

Both endpoints of one line track can be in six different general positions, generating a variation of $6^2 = 36$ different relations, as outlined in Fig. 8. These line tracks are called *Tripartite Line Tracks*, in short $\mathcal{TLT}$, since they are made up of three lines. The i-th relation is accessed by $\mathcal{TLT}(i)$. The medial line is considered to be oriented, for instance, from left to right, i.e. from a to b as exemplified for $\mathcal{TLT}_{36}(1)$.

Consider, for instance, $\mathcal{TLT}_{36}(1)$, $\mathcal{TLT}_{36}(15)$, $\mathcal{TLT}_{36}(22)$, and $\mathcal{TLT}_{36}(36)$. These relations look quite similar since in each case the two endpoints are lying in the same corner-area of the orientation grid. That is, the medial line was originally considered as an oriented vector and in one of these four relations both endpoints are at the top left of the vector, whereas in the other three cases they are at different positions with respect to the medial vector. Similar symmetrical relationships hold for all other relations. But we are interested in non-oriented shape primitives. Hence, in the next paragraph relations are considered with the medial line being oriented not anymore.

4.2 $\mathcal{TLT}_{12}$

The orientation of a shape can be considered with respect to any global frame of reference. The same holds for the parts of any shape. But for shape descriptions

Orientation of the medial line: $\longrightarrow$

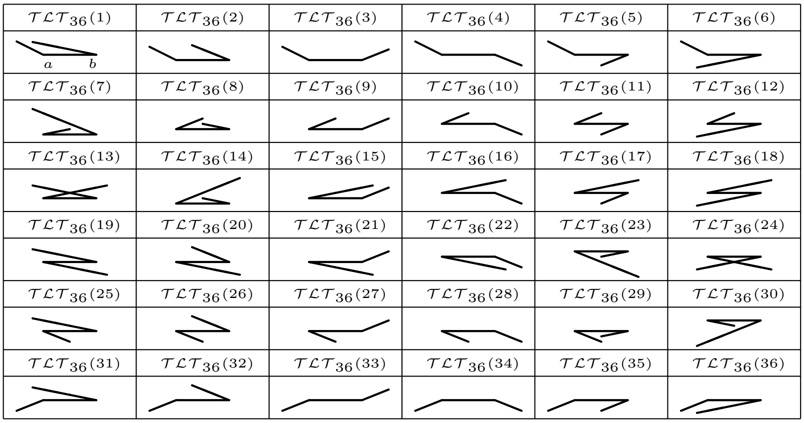

Fig. 8. 36 oriented classes of line tracks with three connected lines

it is sometimes useful to consider in the first place the relationships between parts, and not the orientation of a single part with regard to a global frame of reference. This also holds when we have to cope with incomplete shapes as mentioned in the previous section when we introduced $\mathcal{BLT}_3$. Therefore, it is more expedient to consider parts which are invariant with respect to rotation and reflection. Rotational and reflectional differences are primarily relevant concerning the whole shape.

After removing all symmetrical relations of $\mathcal{TLT}_{36}$ there remain twelve distinguishable relations, as depicted in Fig. 9. The dotted lines outline those areas where the endpoints are allowed to lie in order to satisfy the denoted relation. For simplification, in unambiguous situations we write $\mathcal{TLT}$ instead of $\mathcal{TLT}_{12}$.

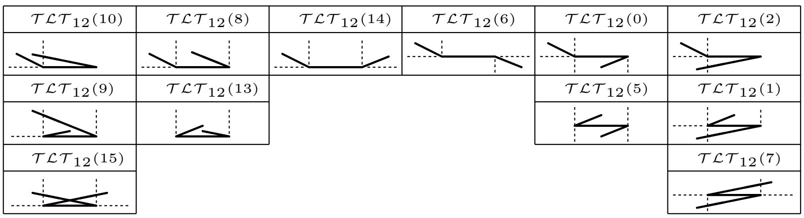

Fig. 9. Twelve non-oriented boundary primitives distinguished by $\mathcal{TLT}_{12}$

4.3 Polygons

A closed polygon with $k \geq 3$ lines is described as a vector of k $\mathcal{TLT}$s, an open polygon as a vector of $k-2$ $\mathcal{TLT}$s:

$$\mathcal{TLT}_{12}(i,j,...), i,j \in \{0,1,2,5,6,7,8,9,10,13,14,15\}.$$

In order to be able to treat each arbitrary polygon with k lines, and not only those consisting of a multiple of three lines, a vector of $\mathcal{TLT}$s describes a polygon in such a way that two consecutive $\mathcal{TLT}$s share two lines. In this way, there exist $\mathcal{TLT}$s which are incompatible. For example, $\mathcal{TLT}_{12}(5)$ and $\mathcal{TLT}_{12}(6)$ cannot be combined since as a combination they would have to share two adjacent lines or one angle, respectively. But the angles of $\mathcal{TLT}_{12}(5)$ are both acute whereas the angles of $\mathcal{TLT}_{12}(6)$ are both obtuse. A compatible combination consists of four lines or two entwined $\mathcal{TLT}$s, respectively.

As for Bipartite Line Tracks, for a given polygon, we choose that description which comes first in the ordering with respect to the $\mathcal{TLT}$ numbers and all possible cyclic permutations.

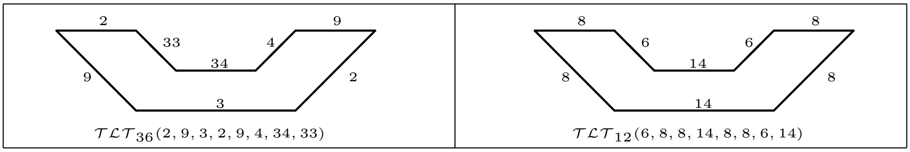

Fig. 10. Example polygon described by Tripartite Line Tracks anticlockwise

Running around the contour of a shape it is reasonable to use $\mathcal{TLT}_{36}$ for a description. In this way, we define an orientation as, for example, anticlockwise regarding the two-dimensional plane, and we are able to distinguish the two sides of the contour. Fig. 10 shows an example shape with both the $\mathcal{TLT}_{12}$ and the $\mathcal{TLT}_{36}$ description. While $\mathcal{TLT}_{36}$ distinguishes both sides of the contour as well as a front-back dichotomy for any part, $\mathcal{TLT}_{12}$ distinguishes parts which are invariant with respect to rotation and reflection. From this follows that similar parts are equally characterised by $\mathcal{TLT}_{12}$, though oriented in different ways with respect to the two-dimensional plane.

4.4 $\mathcal{TLT}$ Neighbourhood and Singular Line Tracks

We define the neighbourhood graph and singular line tracks in a manner similar to that used for $\mathcal{BLT}$s. Fig. 11 shows the $\mathcal{TLT}_{12}$ neighbourhood graph. A *Singular Tripartite Line Track* is referred to as $\mathcal{TLT}(a\text{-}b)$ with one sideline in general position and the other sideline in singular position. In this way, $\mathcal{TLT}(a\text{-}b)$ refers to an edge in the neighbourhood graph, whereby a and b are the nodes which are connected by this edge. Consider the examples in Fig. 12. $\mathcal{TLT}$(13-8-14)

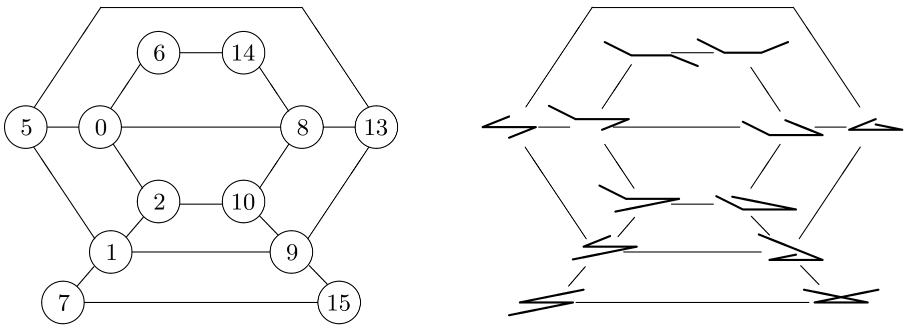

Fig. 11. Left: the neighbourhood graph; Right: example instantiations; the numbers refer to the $\mathcal{TLT}_{12}$-relations

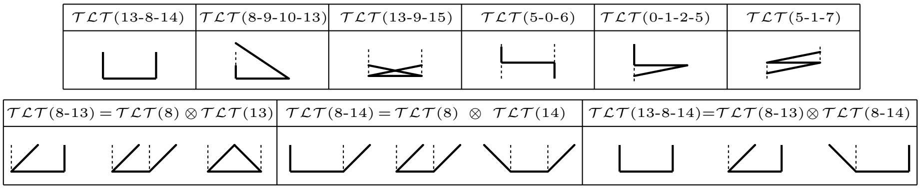

Fig. 12. Upper row: all *Singular Tripartite Line Tracks* with both sidelines in singular position; Lower row: the construction of $\mathcal{TLT}$(13-8-14)

denotes a line track with both sidelines in singular position. Singular line tracks with the endpoints lying on the positions 2, 5, 8, 11, or 14 (see Fig. 1) correspond to degenerated cases. Concerning $\mathcal{TLT}_{12}$ descriptions we are only interested in visually distinguishable line tracks.

5 Reasoning about Shapes

Reasoning tasks for shapes are important by all means: shapes are occluded, nonetheless we want to state something about occluded parts; shapes have to be compared and we are interested in their similarity; shapes may get deformed and physically possible deformation processes have to be described.

5.1 Deformation

Physically possible deformation processes of single $\mathcal{BLT}$s can be described by means of the neighbourhood graph. For instance, $\mathcal{BLT}(1)$ can be directly transformed to $\mathcal{BLT}(2)$ but not to $\mathcal{BLT}(3)$. In order to transform $\mathcal{BLT}(1)$ to $\mathcal{BLT}(3)$ a way through the neighbourhood graph has to be found. More interesting are polygons made up of more than two lines.

We consider change in shape locally, i.e. given any polygon with n lines, a single transformation step consists in changing the position of only one line segment. Such local changes always modify the description of two consecutive $\mathcal{TLT}$s, as exemplified on the right of Fig. 13. On the other hand, regarding the $\mathcal{BLT}$-description only one $\mathcal{BLT}$ is changed. Thus, the $\mathcal{BLT}$-description reflects more appropriately the locality of change, whereas the $\mathcal{TLT}$-description reflects more precisely the effect which a local change has on neighbouring relations.

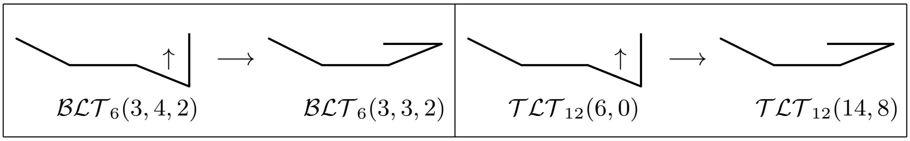

Fig. 13. Changing one line segment changes only one $\mathcal{BLT}$ but two consecutive $\mathcal{TLT}$s; the upward-pointing arrows depict the direction for the deformation process

5.2 Similarity

Deformations and similarity measures are closely related, provided only continuous deformation processes are considered, and this relationship is inversely proportional. The less the deformation applied to any shape, the higher the similarity between the original shape and its deformed complement. Thus, we employ the neighbourhood graph for a measure of similarity likewise describing deformations.

We write $\mathcal{BLT}$ if $\mathcal{BLT}_3$ and $\mathcal{BLT}_6$ are to be treated equally. The distance $\delta(\mathcal{BLT}(i), \mathcal{BLT}(j))$ between $\mathcal{BLT}(i)$ and $\mathcal{BLT}(j)$ is defined as the length of the shortest path between these $\mathcal{BLT}$s in the neighbourhood graph. For their similarity S it holds:

$$S(\mathcal{BLT}(i), \mathcal{BLT}(j)) = \frac{1}{1+\delta(\mathcal{BLT}(i), \mathcal{BLT}(j))}.$$

Two line tracks which represent the same $\mathcal{BLT}$ relation have the highest similarity value of one since their distance is zero. For example, it holds

$$S(\mathcal{BLT}_6(1), \mathcal{BLT}_6(2)) > S(\mathcal{BLT}_6(1), \mathcal{BLT}_6(3)).$$

That is, $\mathcal{BLT}_6(1)$ and $\mathcal{BLT}_6(2)$ are more similar than $\mathcal{BLT}_6(1)$ and $\mathcal{BLT}_6(3)$. In contrast, $\mathcal{BLT}_6(1)$ and $\mathcal{BLT}_6(2)$ are equally similar as $\mathcal{BLT}_6(1)$ and $\mathcal{BLT}_6(6)$:

$$S(\mathcal{BLT}_6(1), \mathcal{BLT}_6(2)) = S(\mathcal{BLT}_6(1), \mathcal{BLT}_6(6)).$$

These relations hold, provided that the similarity between all neighbouring relations in the neighbourhood graph are weighted equally. Traversing through the neighbourhood graph towards a relation $\mathcal{BLT}(i)$, the similarity between $\mathcal{BLT}(i)$ and any passed relation $\mathcal{BLT}(j)$ increases the more in strength the less the distance between $\mathcal{BLT}(i)$ and $\mathcal{BLT}(j)$.

We now consider the similarity between two n-partite polygons

$$P_0 = \mathcal{BLT}(x_0, x_1, ..., x_{n-1}) \text{ and } P_1 = \mathcal{BLT}(y_0, y_1, ..., y_{n-1}).$$

Provided that P_0 and P_1 are aligned in such a way that x_i is aligned to y_j, for their distance we write

$$\delta_{i,j}(P_0, P_1) = \delta_{i,j}(\mathcal{BLT}(x_i, ..., x_{(n-1+i)\, mod\, n}), \mathcal{BLT}(y_j, ..., y_{(n-1+j)\, mod\, n})).$$

The similarity of two polygons, $S(P_0, P_1)$, is based on the most similar alignment of the relations involved. That is, from all possible alignments the one with the smallest distance determines the similarity between two polygons:

$$S(P_0, P_1) = \frac{1}{1+\min_{k=0}^{n-1}\{\delta_{0,k}(P_0, P_1)\}}.$$

While we keep hold of the position of P_0 we consider all possible cyclic permutations of P_1 in order to find the best alignment. The index k represents those cyclic permutations.

For one alignment, the distance between the polygons equals the summation of all distances between two aligned $\mathcal{BLT}$ relations:

$$\delta_{0,k}(\mathcal{BLT}(x_0, ...),\, \mathcal{BLT}(y_k, ...)) = \sum_{i=0}^{n-1} \delta(\mathcal{BLT}(x_i), \mathcal{BLT}(y_{(i+k)\, mod\, n})).$$

For any similarity measurement, the sizes of differently sized polygons have to be taken into account as well. But it depends on the application as to how the difference in size influences the similarity between these polygons.

Weights at the edges of the neighbourhood graph may determine more appropriate similarities between single $\mathcal{BLT}$s since some neighbours may be considered more similar than others. For instance, perceptually it holds

$$S(\mathcal{BLT}_6(1), \mathcal{BLT}_6(2)) > S(\mathcal{BLT}_6(2), \mathcal{BLT}_6(3)).$$

When dealing with singular line tracks singular positions have to be considered like general relations when computing any distance. The same considerations on measuring the similarity between polygons hold for $\mathcal{TLT}$s.

As an example, in Fig. 14, there are all possible convex quadrilaterals made up of $\mathcal{BLT}_3(2)$ and $\mathcal{BLT}_3(3)$, i.e. only the distinction between acute and obtuse angles has been considered along with rectangular angles which correspond to $\mathcal{BLT}_3$(2-3). Even so, we are able to distinguish many different quadrilaterals. However, we cannot distinguish some of those simple convex polygons from other concave polygons. For those distinctions we have to use $\mathcal{BLT}_6$. The relationship between deformation and similarity is illustrated in Fig. 14. Two quadrilaterals are connected if a local deformation step transforms one of them into the other one. Simultaneously, two quadrilaterals are more similar regarding our similarity measurement the less deformation steps are necessary for such transformations.

[7] demonstrates how to distinguish different object categories by describing salient contour parts with the aid of $\mathcal{TLT}_{12}$. The $\mathcal{TLT}_{12}$ neighbourhood graph is used as a measure of similarity between an object-instance and possible categories. In this way, even the complex shapes of similar natural objects such as

cats and dogs, which are represented in a stylised manner by simple polygons, can be discriminated. Each category is adequately described by typical tuples and triples of $\mathcal{TLT}$ relations. Tuples or triples of $\mathcal{TLT}$ relations at special positions, characterise object categories and constitute salient object-specific spatial structures. When replacing $\mathcal{TLT}$s in such structures with neighbouring relations, similar spatial structures are obtainable.

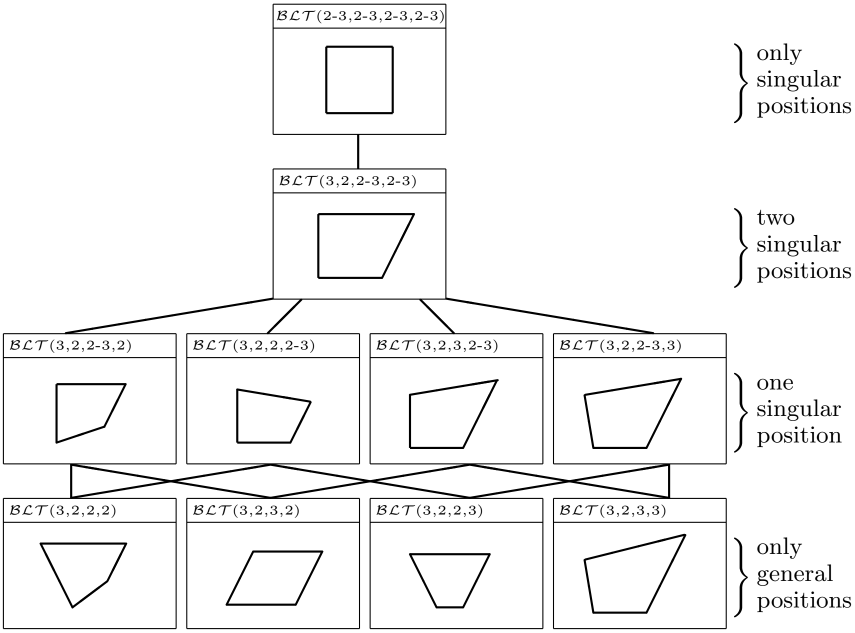

Fig. 14. Simple convex quadrilaterals described by $\mathcal{BLT}_3$; for comparisons the anticlockwise encodation starts always with the bottom line

5.3 Occlusion

Among other reasoning tasks, [5] considered the problem of specifying a short cut from a position a to another position c, given the position of c with respect to the path $a \to b$. More formally, the idea is to deduce a position b relative to a vector $a \to c$, provided that position c with respect to the vector $a \to b$ is known. This task can be cast into the following shape reasoning problem:

> We see a partly occluded object and notice two arbitrary parts, a and b, as illustrated on the left of Fig. 15. The object moves and suddenly one of these two parts is occluded, for example, b as shown on the right of Fig. 15. Then, we would like to know where the occluded part is now, with respect to a and another visible part c.

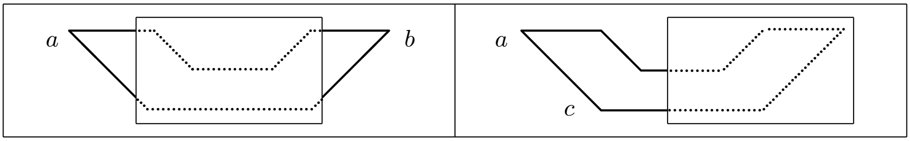

Fig. 15. A polygon being partly occluded in two different ways, at two different times

Table 1. Given (c w.r.t. $a \to b$) (left column) derive (b w.r.t. $a \to c$) (middle column), and derive (c w.r.t. $b \to a$) (last column) - encoded by $\mathcal{BLT}_6$

c w.r.t. $a \to b$	b w.r.t. $a \to c$	c w.r.t. $b \to a$
$\mathcal{BLT}$(1)	$\mathcal{BLT}$(6)	$\mathcal{BLT}$(4)
$\mathcal{BLT}$(2)	$\mathcal{BLT}$(4) $\vee$ $\mathcal{BLT}$(4-5) $\vee$ $\mathcal{BLT}$(5)	$\mathcal{BLT}$(5)
$\mathcal{BLT}$(3)	$\mathcal{BLT}$(5)	$\mathcal{BLT}$(6)
$\mathcal{BLT}$(4)	$\mathcal{BLT}$(3)	$\mathcal{BLT}$(1)
$\mathcal{BLT}$(5)	$\mathcal{BLT}$(2) $\vee$ $\mathcal{BLT}$(2-3) $\vee$ $\mathcal{BLT}$(3)	$\mathcal{BLT}$(2)
$\mathcal{BLT}$(6)	$\mathcal{BLT}$(1)	$\mathcal{BLT}$(3)
$\mathcal{BLT}$(1-2)	$\mathcal{BLT}$(5-6)	$\mathcal{BLT}$(4-5)
$\mathcal{BLT}$(2-3)	$\mathcal{BLT}$(4-5)	$\mathcal{BLT}$(5-6)
$\mathcal{BLT}$(4-5)	$\mathcal{BLT}$(2-3)	$\mathcal{BLT}$(1-2)
$\mathcal{BLT}$(5-6)	$\mathcal{BLT}$(1-2)	$\mathcal{BLT}$(2-3)
$\mathcal{BLT}$(3-4)	$\mathcal{BLT}$(2-5)	$\mathcal{BLT}$(1-6)
$\mathcal{BLT}$(2-3-4-5)	$\mathcal{BLT}$(2-3-4-5)	$\mathcal{BLT}$(1-2-5-6)
$\mathcal{BLT}$(2-5)	$\mathcal{BLT}$(3-4)	$\mathcal{BLT}$(2-5)
$\mathcal{BLT}$(1-2-5-6)	every position is possible	$\mathcal{BLT}$(2-3-4-5)
$\mathcal{BLT}$(1-6)	$\mathcal{BLT}$(1-6)	$\mathcal{BLT}$(3-4)

We assume that one is able to retain coarsely the vector $a \to b$ after b has been occluded, at least for a short moment. Thus, when realising c, it can be described with respect to $a \to b$, and this relationship is easier to keep in mind than the single position of b, which gradually diminishes. But later, it is absolutely possible to state where the position of b is with respect to the vector $a \to c$. In this way, Table 1 allows us to deduce the position of b regarding $a \to c$, given c with respect to $a \to b$.

For $\mathcal{BLT}$(2) and $\mathcal{BLT}$(5), there exist three possible positions of b with respect to $a \to c$. Hence, these relations are not unambiguously deducible. Consider $\mathcal{BLT}$(4) and $\mathcal{BLT}$(5). These two cases are different since the position of b in the former case is doubtlessly $\mathcal{BLT}$(3). In contrast, the situation is uncertain for $\mathcal{BLT}$(5) since both positions, $\mathcal{BLT}$(2) and $\mathcal{BLT}$(3), are possible. This uncertainty arises because the position in question may be near the transition between $\mathcal{BLT}$(2) and $\mathcal{BLT}$(3). For $\mathcal{BLT}$(1-2-5-6) the situation is highly ambiguous since every position is a possible position for b.

As an example consider again Fig. 15. When the object moves to the right, the position of c with respect to $a \to b$ is $\mathcal{BLT}$(5). When we rotate Fig. 15 in order to describe b with respect to $a \to c$, we learn that b is somewhere on the left-middle or left front of the vector $a \rightarrow c$, and thus we obtain the relations $\mathcal{BLT}(2)$, $\mathcal{BLT}(2\text{--}3)$, and $\mathcal{BLT}(3)$.

It seems inappropriate to rotate the image first in order to describe the position of the occluded part b. This rotation is necessary because c is below a regarding the image plane, and because we want to describe b with respect to $a \rightarrow c$. It would be more appropriate to describe b with respect to $c \rightarrow a$. For such cases, we describe the inverse relation in the rightmost column of Table 1 and we are then able to derive b with respect to $a \rightarrow c$. Freksa and Zimmermann show that in contrast to the first operation the inversion operation always yields precise results. Let us consider how this especially concerns singular positions. We treat singular positions as positions about which uncertainty exists regarding neighbouring general positions. The inverse relations of singular positions correspond to such uncertain positions, too. At this, imprecision and uncertainty must not be confused. The inversion operation is precise in the sense that we do not have any disjunction in the right column of Table 1, i.e. any inverse relation is unambiguously deducible regardless of whether it corresponds to a singular position or a general position.

6 Related Work

There exist a vast amount of quantitative shape descriptions but only a few qualitative approaches. In [2] there is a short overview of some qualitative shape descriptions, and we will only mention those which are mostly related to our own approach.

Closely related to our approach is [10], who describes polygons by considering triangle orientations of vertices. Not restricted to polygons, but also confined to two-dimensional outlines is the approach of [6]. They consider seven different curvature types and propose a grammar for their combination. In contrast, [1] investigates a topological approach for shape descriptions. He distinguishes different concave regions by considering the notion of connection of regions and their convex hulls. [8] shows that $\mathcal{TLT}$s are more expressive than the approach of [10], and further comparisons with other descriptions have to be accomplished.

7 Discussion

Four sets of relations have been presented, namely $\mathcal{BLT}_3$, $\mathcal{BLT}_6$, $\mathcal{TLT}_{12}$, and $\mathcal{TLT}_{36}$. The relationships between these sets have to be investigated more thoroughly. For now, a kind of cooperation between these relations can be reconciled with a strategy of least commitment, which is frequently recommended in vision. This means that any description is only allowed to be as precise as the available conditions allow. Particularly shape descriptions in earlier stages of object recognition should not be over-constrained. Fuzziness and incompleteness require shape descriptions which are, among other things, invariant with respect to orientation. For instance, it may not be clear how single contour parts are oriented with respect to a shape's region, when the region could not be detected because of noisy image data. In this case, an invariant description like $\mathcal{TLT}_{12}$ may be useful, and for less complex shapes, even $\mathcal{BLT}_3$. In contrast, $\mathcal{BLT}_6$ and $\mathcal{TLT}_{36}$ are more appropriate for descriptions in later stages of object recognition. For example, when the distinction between figure and ground is already established, and hence the orientation of any contour part. Cutting matter short, the more knowledge obtained, the more constrained is any description allowed to be.

From the data acquisition point of view, it seems appropriate to deal with polygonal structures like in the present paper. 58% of almost one hundred papers reviewed by [3], apply thinning techniques and polygonal approximations to shapes. Moreover, there exist a lot of algorithms for polygonal approximations and [9] even develops several measures to assess the stability of such algorithms.

Generally speaking, we demonstrated how qualitative spatial reasoning techniques can be adopted for reasoning about shapes. This concerns particularly orientation information applied in the context of navigation. The relationship between navigation and shape is quite obvious since outlines of shapes and trajectories which are related to navigation, are identical from a geometrical point of view. As one can imagine, other spatial reasoning methods may be similarly suitable in order to better manage the complexity of shapes in the future.

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