Global Feature Schemes for Qualitative Shape Descriptions

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Abstract

Qualitative shape descriptions are important in a number of fields in AI, such as in qualitative reasoning, and especially in robotics and vision. Current approaches are confined to describing local features while walking along the shapes' boundaries. Shapes exist, however, which cannot be distinguished by these methods even though there are obvious distinctions between them. We shall introduce the notion of a global feature scheme in order to compensate for the shortcomings of present techniques. This approach is then applied to a class of shapes which have previously been shown to be difficult to distinguish, and proves capable of telling them apart. The method is based on a representation of intersection-free relations which complements existing qualitative representations for which intersections are virtually fundamental.[1]

1 Introduction

Qualitative shape descriptions are useful for symbolic shape reasoning tasks such as when studying the behaviour of objects, their motion in space, interactions with other objects, and changes in shape [Gottfried, 2003b]. In order to manage the complexity of shapes it is meaningful to confine oneself to simply connected shapes, as when considering the boundaries of objects. Boundaries are particularly important since they constrain how objects are related to other objects and their environment.

Qualitative boundary based approaches which have been discussed so far are restricted to *local feature schemes*, as they are referred to by [Meathrel and Galton, 2000]. Such approaches record features of the boundary while walking along it. Those which have been devised most recently include [Gottfried, 2003a], [Gottfried, 2003b], [Meathrel and Galton, 2001], [Meathrel and Galton, 2000], [Galton and Meathrel, 1999], [Schlieder, 1996], and [Jungert, 1993].

Galton and Meathrel base their approach on a discretisation of tangent bearing and curvature, considering the rate of change of curvature, and they give a set of tokens from which higher-level tokens can be derived. By contrast, all other approaches rely on polygonal approximations of the underlying boundaries. There are several reasons for this. Most importantly, the discrete space used in computer vision inherently deals with polygons; depending on the application at hand outlines can be approximated at a number of different granularity levels when using polygons; frequently, polygons are directly given (when recording the path of a navigating system the position of which is measured at intervals, for example); polygons provide concise representations of even highly complex outlines, forming an appropriate basis for the qualitative characterisation of shapes from both the local and global points of view. However, regardless of whether they use polygons or not, local feature schemes suffer from a lack of information about relations between distant parts of the boundary. This is the reason why it is necessary to introduce the notion of a *global feature scheme*.

2 Local versus global features

While local feature schemes consider the ordering of features around the boundary, there are additional properties which derive from the circular ordering of features. Among other things, whether two features are directly adjacent to each other and, if not, what kinds of features exist between them. If they are not adjacent, then numerous different relationships between them (in terms of their position and orientation) may be possible. Such relationships may be distinctively different between classes of shapes that local feature schemes are unable to tell apart. For any global feature scheme we stipulate that a number of such *disconnection relations* between distant features or parts are to be defined. What distinguishes global feature schemes is that they take into account such non-local relations. In this case, therefore, we are using the term *global* in the sense of *non-local* rather than in the sense of *everything together* or *embracing the whole*.

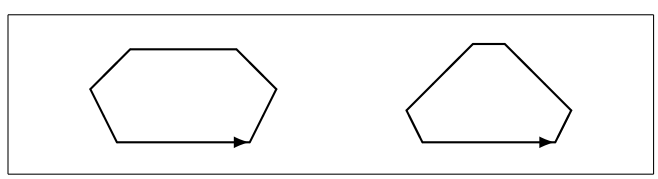

Figure 1: Two convex polygons

[1] In: H. W. Guesgen, Ch. Freksa, G. Ligozat (Ed.), IJCAI-05 WS on spatial and temporal reasoning. Edinburgh, Scotland, 2005.

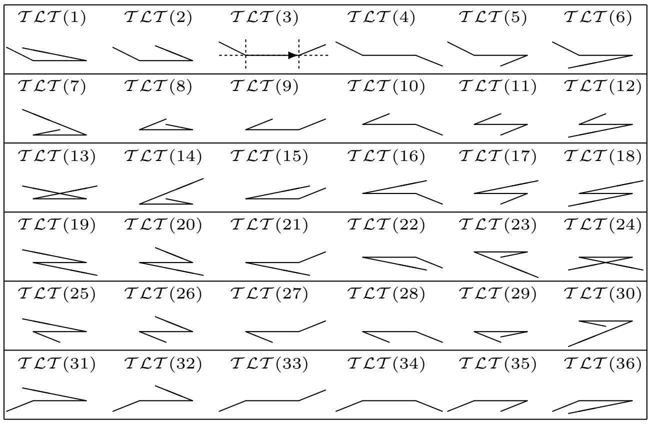

Figure 2: 36 distinguishable classes of line track arrangements with three connected lines, oriented from left to right; in the present context $\mathcal{TLT}(13)$ and $\mathcal{TLT}(24)$ cannot occur

The shortcomings of local feature schemes can be illustrated by the polygons in Fig. 1. Those qualitative local feature schemes which have been proposed so far are not capable of distinguishing these two polygons; for both polygons it holds that:

Gottfried (a)	each primitive is described by $\mathcal{TLT}(3)$
Gottfried (b)	each primitive is described by $\mathcal{BLT}(3)$
Galton, Meathrel	each curvature type is described by "/>"
Schlieder	all triangle orientations are positive
Jungert	all vertices are convex and obtuse

Another approach worth mentioning in this context distinguishes shapes by their concavities which are recursively described by their own concavities until only convex parts remain [Cohn, 1995]. This approach, like those mentioned previously, is capable of distinguishing convex from concave objects but none of them can distinguish two convex polygons, such as those in Fig. 1; though [Jungert, 1993] distinguishes at least some convex polygons by additionally considering whether adjacent angles are equal or not, and [Gottfried, 2003a] distinguishes a number of different convex quadrilaterals by taking into account three qualitative primitives, namely $\mathcal{TLT}(2)$, $\mathcal{TLT}(3)$, and $\mathcal{TLT}(8)$ in Fig. 2. Therefore, we will investigate this last approach more closely.

2.1 Tripartite line tracks

A local feature scheme for polygons is provided by *tripartite line tracks* ($\mathcal{TLT}$ for short) [Gottfried, 2003a] and [Gottfried, 2003b]. It characterises polygons by considering the arrangement of a polygonal line together with its predecessor and successor, i.e. by considering its local context. Fig. 2 shows these relations. A polygon is characterised by determining for each line segment its $\mathcal{TLT}$.

Tripartite line tracks are based on the orientation grid [Zimmermann and Freksa, 1996] which is defined by a vector. It partitions the plane into six regions, as shown by the dotted lines for $\mathcal{TLT}(3)$ in Fig. 2. When applied to three connected lines it is, for instance, possible to distinguish acute and obtuse angles. $\mathcal{TLT}$s consider only the special case of arrangements of lines which are adjacent in a polygon, but the orientation grid can be used in a more general way, describing the relative position between two distant lines, one of these defining the orientation grid. Such arrangements between two lines have been referred to as *bipartite arrangements* [Gottfried, 2004a].

2.2 Bipartite arrangements

Disconnection relations for the purpose of supervising traffic scenarios have been introduced in [Gottfried, 2004a]. A number of jointly exhaustive and pairwise disjoint *disconnection relations*, $\mathcal{BA}$, are proposed which allow for the description of extended and directed objects, or, abstractly speaking, this approach allows for the representation and reasoning about arrangements of intervals in the plane. The relations of $\mathcal{BA}$ are shown in Fig. 3. They form a relation algebra whose operations are defined in [Gottfried, 2004a]. Note that these relations are jointly exhaustive since they represent arrangements between objects which cannot intersect, for instance, vehicles in the street, or line segments of the polygonal outline of the silhouette of an object; or, more generally, a number of rigid objects in the plane among which only one topological relation can hold, namely disconnectedness.

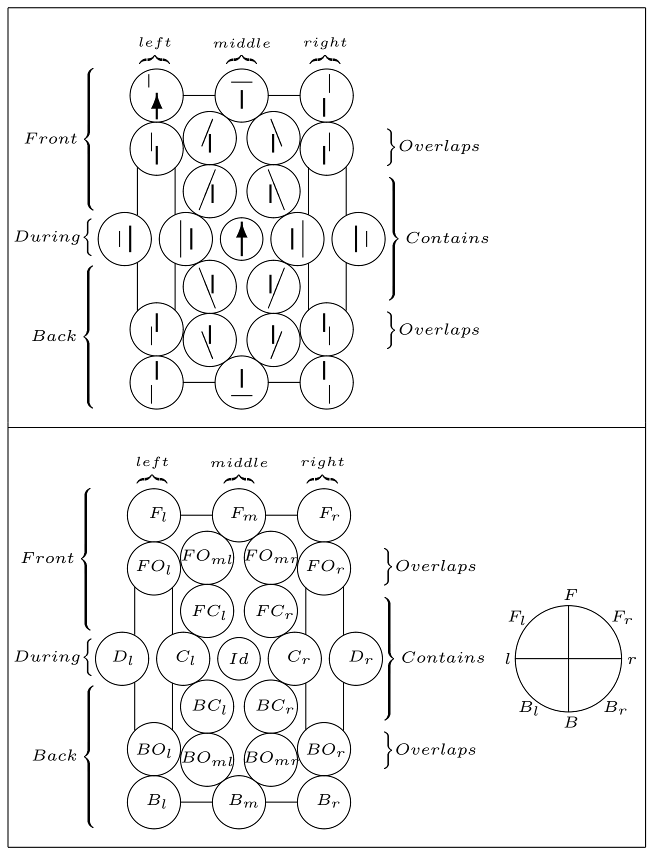

Figure 3: Top: Interval relations embedded in two dimensions, the vertical reference interval being displayed bold; Bottom: Abbreviations and the orientation variations (right)

3 A global feature scheme

If one line of a polygon is made the basis of the orientation grid, the position and orientation of every other line can be described relative to it. In this way, the global context of a polygonal line is considered, whereas $\mathcal{TLT}$s are restricted to the local context. For the time being, we will confine the description to positions. Then, for a polygon with n lines we obtain a list of relations to which we refer to as the *course* of reference segment x, in short $C(x)$:

$$C(x) \equiv (x_{y_1}, .., x_{y_n})\ x_{y_i} \in \mathcal{BA}, i = 1, .., n$$

with x_{y_1} meaning that line y_1 is described with respect to reference line x. In particular, it holds that $x_x = Id$.

Having a polygon with n lines there exist n courses, each one comprising n bipartite relations. But arrangements of adjacent lines form local properties which are already described by local feature schemes. Thus, for each course we omit those two bipartite relations which are adjacent to the reference line; each course then comprising only disconnection relations while $\mathcal{TLT}s$ describe the connection relations. Writing down all courses, one below the other, for a closed polygon with five lines the following matrix is obtained:

$$M = \begin{array}{ccccc} Id & - & u_w & u_x & - \\ - & Id & - & v_x & v_y \\ w_u & - & Id & - & w_y \\ x_u & x_v & - & Id & - \\ - & y_v & y_w & - & Id \end{array}$$

We define the orientation of the polygon always to be anticlockwise and start the description with an arbitrary line. As a consequence, different matrices can be obtained for identical closed polygons. But, if necessary, any matrix can be converted into another equivalent one by means of a cyclic swapping of all rows. For a closed polygon with n lines our global feature scheme comprises $n^2 - 3n$ relations, and for open polygons $n^2 - 3n + 2$ relations.

One important advantage of this approach is its ability to cope with indeterminate boundaries: Incompleteness arises if there are gaps in the boundary; nonetheless, all relations between the discernible segments can be used in the described way; this allows to describe the incomplete shape at least partly. Imprecision arises as a consequence of weak sensory information; it is dealt with implicitly by the coarseness of $\mathcal{TLT}$s and $\mathcal{BA}$-relations.

Overall, the strength of these techniques is that the approach is particularly well suited for dealing with coarse shape information. Once precise shape details matter it may be appropriate to apply quantitative methods instead. By contrast to a local feature scheme, which requires only linear time complexity for obtaining the description, the complexity of computation of the matrix is quadratic — a natural trade-off between expressiveness and complexity.

4 Reconciling local and global features

How do local and global features work together? While adjacent lines are described by $\mathcal{TLT}$s non-adjacent lines are described by $\mathcal{BA}$s. For the polygon on the left hand side of Fig. 1 we obtain the following matrix:

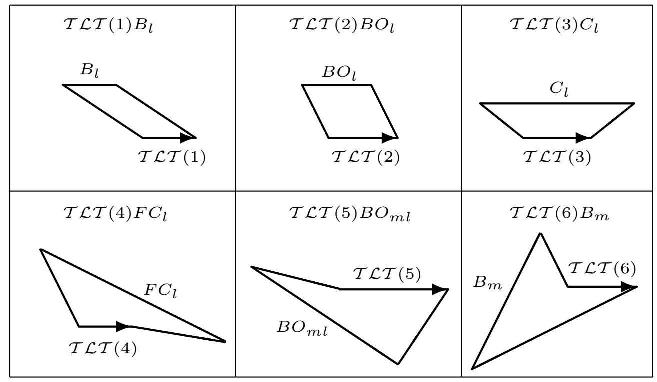

Figure 4: The first six quadrilaterals

$$\begin{array}{cccccc} 3 & - & FO_l & D_l & BO_l & - \\ - & 3 & - & FO_l & BO_l & B_l \\ C_l & - & 3 & - & F_l & F_l \\ C_l & B_l & - & 3 & - & F_l \\ C_l & B_l & B_l & - & 3 & - \\ - & F_l & FO_l & BO_l & - & 3 \end{array}$$

For the polygon on the right hand side of Fig. 1 we get:

$$\begin{array}{cccccc} 3 & - & FO_l & D_l & BO_l & - \\ - & 3 & - & F_l & C_l & B_l \\ BO_l & - & 3 & - & F_l & FO_l \\ C_l & B_l & - & 3 & - & F_l \\ FO_l & BO_l & B_l & - & 3 & - \\ - & F_l & C_l & B_l & - & 3 \end{array}$$

The diagonal shows that the polygons are locally equal since they are made up of the same $\mathcal{TLT}$s. That is to say, while the diagonal represents a local feature scheme the rest of the matrix represents a global feature scheme. The latter shows in what kinds of aspects the two polygons differ. From the point of view of the first line, i.e. the horizontal line at the bottom which is defined to be oriented from left to right as indicated by the vector, the polygons are equal. But when traversing anticlockwise to the next reference line the two polygons show some differences. While for the left polygon the relations FO_l and BO_l hold (first matrix second row), for the right polygon F_l and C_l hold at the corresponding positions (second matrix second row). These differences are related to the way in which differences in length induce differences in positional relations. There are similar differences for the courses of the other reference lines which cannot be described by means of local feature schemes. Fig. 4 shows some examples of polygons which are distinguishable using our approach.

5 The shapes of Galton and Meathrel

[Galton and Meathrel, 1999] show some limitations of their own approach. The five outlines in the first row of Fig. 5 are indistinguishable by their method and by those generalisations of [Meathrel and Galton, 2000] and [Meathrel and Galton, 2001], since neither of them deals with global features. They argue that their approach could be extended in order to allow for these distinctions in two different ways. Firstly, each symbol could be annotated with indices such as U, D, L, R for upward pointing, downward pointing, left pointing, and right pointing. Unfortunately, this means their approach ceases to be rotation invariant; an important property when using such a shape description for indexing image databases with arbitrarily aligned objects. Secondly, the symbols could be annotated by indices denoting their relative length, and this could be accordingly done for angles. This is an obvious possibility, and has also been suggested by others (such as [Jungert, 1993]). But this approach has the disadvantage that measurements are required in order to obtain quantitative information for the purpose of comparing the size of angles or the length of curves or sides. As Galton and Meathrel already obtain their description by measuring the tangent bearing, curvature, and rate of change of curvature while walking along an outline, this causes no problem for them. However, it demonstrates another important difference of our approach. We have no need of any quantitative extensions, but can base our description entirely on coarse positional information. This kind of information is robustly obtainable from image data and sensory data; in [Gottfried, 2004a] it is even shown how to compensate for incomplete, imprecise, and erroneous information while using the algebra of $\mathcal{BA}$. By positional information alone it is possible to distinguish the shapes in Fig. 5, and this kind of positional information needs neither any quantitative extension nor an external reference system but describes the outlines in a self-referring manner. This is the reason why the description is rotation invariant.

The proposed combined feature scheme distinguishes the shapes in Fig. 5 as follows. The arrangements of notches in the outlines are obviously different. These differences leap to the eye and they should therefore be taken into account in a qualitative description. In other words, each shape is made up of two convex parts which are interrupted by two concavities. The convex and concave parts can be identified by local features alone. What then matters is how the convex and concave parts are related to each other by positional information, that is, from the global perspective.

5.1 Shape approximations

For the purpose of applying our feature scheme to these shapes they have to be approximated by polygons. Using the approach of [Douglas and Peucker, 1973] we obtained shape approximations at the same level of granularity for all the shapes, in the sense that the same error threshold is used (see the second row in Fig. 5). This error threshold describes the difference between the approximating polygon and the original outline (bottom row in Fig. 5).

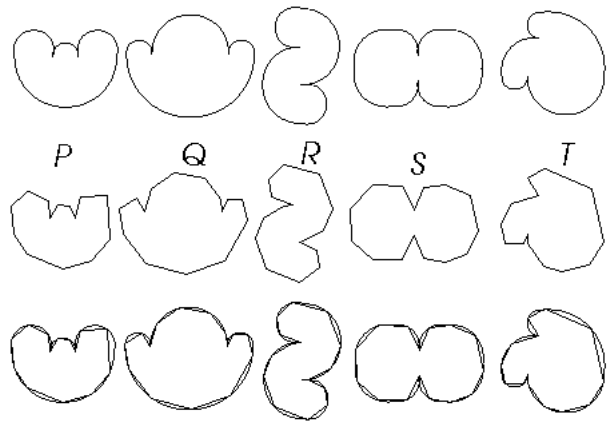

Figure 5: Top row: distinct outlines all described by $\supset\prec\supset\prec$, Fig. 4 from [Galton and Meathrel, 1999], Middle row: polygonal approximations, Bottom row: error difference

5.2 Distinguishing concavities and convexities

Concavities and convexities can be distinguished locally by $\mathcal{TLT}$s, i.e. the set of all $\mathcal{TLT}$s is divided up into two equivalence classes, one containing only concave and the other one only convex $\mathcal{TLT}$s. In all shapes both concavities are described by $\mathcal{TLT}(5, 27)$ with the exception of Q and T which instead have one concavity with $\mathcal{TLT}(5, 26)$ and $\mathcal{TLT}(11, 27)$, respectively — but note that $\mathcal{TLT}(26)$ is conceptual neighbour of $\mathcal{TLT}(27)$ and $\mathcal{TLT}(11)$ of $\mathcal{TLT}(5)$, meaning that these primitives have significant similarities (see Fig. 2 and [Gottfried, 2003a]).

All convex parts are described by chains of $\mathcal{TLT}(3)$-relations, with the exception of Q and T which also have a $\mathcal{TLT}(2)$ and $\mathcal{TLT}(9)$-relation, respectively, which are both conceptual neighbours of $\mathcal{TLT}(3)$ (see Fig. 2) and which are also both convex.

5.3 Positions between parts

For each shape the relative position between the two concavities can be described by taking the lines with the local context of $\mathcal{TLT}(5)$ in place of the concavity (and in one of the two cases of polygon T by $\mathcal{TLT}(11)$). We denote one of the concavities of each shape by x and the other by y. Their relative position is as follows:

$$\begin{array}{llcl}
P: & x_y = FO_r & \wedge & y_x = BO_l \\
Q: & x_y = F_r & \wedge & y_x = BO_l \\
R: & x_y = F_r & \wedge & y_x = F_r \\
S: & x_y = F_l & \wedge & y_x = F_l \\
T: & x_y = F_r & \wedge & y_x = F_l
\end{array}$$

Distinguishing P, Q, T from S, T

By means of these relations we may put P, Q, and T in one equivalence class, and R and S in another one. In the first class all concavities are approximately near each other as being on the same side, as far as one is disposed to consider side-like sections in a roundish shape. This derives from their positions: one concavity is left of the other one, whereas the other one is right of the former one. By contrast, for R and

S the concavities are opposite to each other, i.e. they are on different sides. This can be derived from their relative positions which are for both points of views equal regarding the left-right dichotomy. In R each concavity is relatively right of the other one, and in S they are left of each other.

Distinguishing R from S

R and S can be distinguished as follows. The concavities of R are somehow shifted relative to each other, whereas those of S are opposite to each other, they almost point to each other. Denoting the first $\mathcal{TLT}(27)$-component by x' and the second one by y' (compare Fig. 6) for R it holds that

$$\begin{pmatrix} x_y & x_{y'} \\ x'_y & x'_{y'} \end{pmatrix} = \begin{pmatrix} F_r & F_r \\ B_l & B_l \end{pmatrix}$$

$$\begin{pmatrix} y_x & y_{x'} \\ y'_x & y'_{x'} \end{pmatrix} = \begin{pmatrix} F_r & F_r \\ BO_l & BO_l \end{pmatrix}$$

The changes of *left* and *right* between the rows reflect the shift of concavities. By contrast for S it holds that

$$\begin{pmatrix} x_y & x_{y'} \\ x'_y & x'_{y'} \end{pmatrix} = \begin{pmatrix} F_l & F_l \\ B_l & B_l \end{pmatrix}$$

$$\begin{pmatrix} y_x & y_{x'} \\ y'_x & y'_{x'} \end{pmatrix} = \begin{pmatrix} F_l & F_l \\ B_l & B_l \end{pmatrix}$$

In this case there is no change between *left* and *right* (compare the orientations of the components of the concavities).

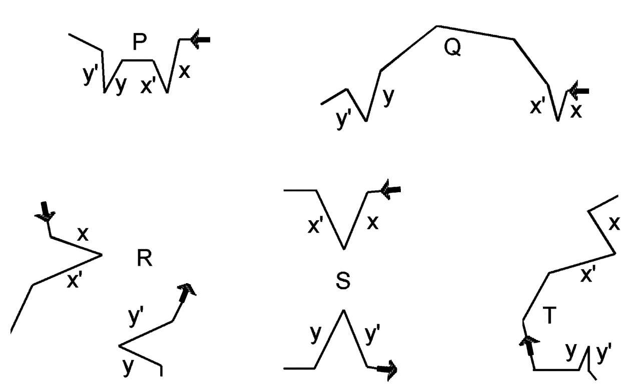

Figure 6: Relations between concavities (x, x') and (y, y')

Taking these relations for P and Q they cannot be distinguished. But at least T is slightly different from P and Q regarding these relations. This corresponds to the asymmetric curvature of T whereas there is an arc-like convexity in P and Q.

Distinguishing Q from T

Q and T are quite similar as both have a small convex part which overhangs relative to the rest of the outline. This can be comprehended very well if rotating T so that its small convexity is also at the top regarding the picture-plane. The difference in curvature of these convex parts can be derived by the relations which characterise these convexities. Half of them in T are overlap-relations whereas there are only two overlap-relations in the corresponding, less curved, part of Q.

Distinguishing P from Q, and P from T

Finally, P and Q can be distinguished by the central convex parts at the top. In P this part is smaller than the one in Q, i.e. it is approximated by fewer lines. Additionally, in P this part is inside the convex hull of the outlines whereas it is lying on the convex hull in Q. This can be derived from its position relative to the rest of the polygon. From the viewpoint of the upper part of the convexity in Q the rest of the outline is completely left of it, indicating that this part lies on the convex hull of the shape. This does not hold for the corresponding part in P which is slightly imbedded in the outline. The same considerations allow to distinguish P and T.

5.4 Summary

We are able to distinguish the shapes of Fig. 5 though they are quite similar in that they all comprise two equally sized concavities and two convexities. We identified those characteristics which are obvious from the point of view of perception, i.e. we take into account the relative positions of concave parts, the difference in curvature of convex parts, and the relative positions of convex parts regarding the whole outline.

Although this scenario consists of only a few examples it is sufficient to demonstrate the usefulness of our approach:

1. The scenario shows how the approach is able to cope with sets of similar shapes which differ only in the arrangement of concave and convex parts. By contrast, other qualitative approaches treat such shapes to be equal in as much as they are all made up of the same number of concavities and convexities. There are two things to note: (i) concavities can always be described locally by $\mathcal{TLT}$s, therefore the approach can be applied to arbitrary sets of shapes; (ii) the decisive matrix entries can be derived from the diagonal which describes what kind of role a line plays regarding concavities and convexities.
2. The scenario shows that the approach can be regarded as an important extension to Galton and Meathrel since we can distinguish shapes which they cannot distinguish, and we do this in a purely qualitative and self-referring way, that is, this technique is robust and rotation invariant, respectively.
3. As previous methods show, local feature schemes suffer from a lack of information regarding disconnected parts. Here, we have demonstrated both how local feature schemes could be extended using global features, and that disconnection relations (defined in $\mathcal{BA}$) can be a useful basis for global feature schemes.

6 Conclusions

We introduced the notion of a global feature scheme which complements those local feature schemes which have been proposed during the last decade. We developed a global feature scheme designed to complement one of the local feature schemes which have been proven to be useful in qualitatively characterising polygons. In particular, the new approach allows to identify shape features which are not distinguishable by means of local feature schemes. This approach clearly pertains to the class of boundary based techniques, which are significant because boundaries constrain how objects relate to other objects and their environment.

A crucial point of our approach concerns the availability of a great variety of positional relations. This requires an appropriate way of defining which relations matter in a particular context. In the example given it proved to be useful to describe relative positions between parts, i.e. between concavities and also between convex parts, and the relation of these to the outline as a whole.

An important issue concerns the question of how to choose an appropriate granularity level for polygonal approximations and another important issue concerns the robustness of the feature scheme with respect to different granularity levels. A parameter which is important for determining an appropriate granularity level is the compactness of objects. The less compact an object is, the finer the granularity level of the polygon has to be, so that no details get lost. In our case the objects are all similarly compact. Therefore, a single granularity level could be used for all shapes.

In [Gottfried, 2003a] it is shown that $\mathcal{TLT}$s are already more expressive than [Schlieder, 1996]. [Gottfried, 2005] shows that $\mathcal{BLT}$s, which form a generalisation of $\mathcal{TLT}$s, are equally expressive as [Jungert, 1993]. On the other hand [Jungert, 1993] is less expressive than our global feature scheme. Comparisons to further approaches are currently under examination.

Our shape description can be extended in several ways. Among other things, the orientations of the lines involved could be considered, for example, in order to distinguish $F_l^{F_l}$ from $F_l^{F_r}$, and as a consequence, make possible further distinctions. Notably, we have not addressed the issue on singular relations yet. Such relations arise when, for instance, lines are precisely aligned and equal in length. For now, whenever we encounter singular relations we can assign them to one of their adjacent relations in the neighbourhood graph; in doing so we must select a neighbour which is itself in the neighbourhood of the next but one relation. A thorough discussion of singularities can be found in [Gottfried, 2004b].

Our representation of intersection-free relations complements other qualitative representations for which intersections are virtually fundamental ([Egenhofer and Franzosa, 1991], [Randell *et al.*, 1992]). But there are actually areas which are restricted to intersection-free relations: when modelling the behaviour of motorists and cyclists in the street, or in qualitative shape descriptions when being confined to two-dimensions, especially when using the polygonal boundaries of objects; this is either of interest when only coarse shape information is available or for the purpose of prescreening large image databases. Finally, the described method has been used for characterising outlines of graphical queries for the purpose of searching for images in databases [Gottfried, 2005].

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